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Mergers and acquisitions strategy under imperfect information: A mixed payment model

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Abstract: This paper develops a dynamic model of a joint takeover to determine the timing, acquisition premiums, and terms. The model incorporates imperfect information and the strategy by solving a Markov perfect Nash equilibrium. The results predict that the bidder will make a high cash payment to the target to gain high post-merger management control. The abnormal return to the participating firm can be positive or negative due to asymmetric information. In addition, the model relates the acquisition premium payment and the merger threshold to the growth rate, volatility, and correlation coefficient of the bidder and target.

Keywords: Sharing-rule, Real option, Takeover, Acquisition premium, Synergy, Bargain power

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1 Introduction

Generally, enterprises have two ways to grow: one is by expanding production, which is internal reinvestment; and another is by acquisition and reorganization. Acquisitions will create synergies from cost savings, customer expansion, or other financial benefits resulting from the cooperation of two firms.

The optimal timing and post-merger ownership structure are two keys in the analysis of merger and acquisition strategies. Several famous studies analyzed the timing and terms of mergers and acquisitions. Lambrecht [11] studies the timing and terms of mergers motivated by economies of scale and shows that merger activities are positively correlated with markets, meaning that firms are willing to merge during economic expansions. Thijssen [21] extends Lambrecht [11] into a two uncertain conditions model and optimizes the timing for considering both efficiency gains and diversification benefits. The results show that mergers and acquisitions will during both economic upswings and downswings. Bernile, Lyandres and Zhdanov [4], Hackbarth and Morellec [10], and Lambrecht and Myers [12] also develop models of optimal timing. Hackbarth and Miao [9] develop a joint model of oligopolistic industries that determines the industry's product equilibrium and the timing and terms of takeovers. The result shows the relationship between the return from mergers and company size. In terms of firm-size analysis, Moeller, Schlingemann, and Stulz [16] show the firm size effect in acquisition announcement returns.

There are two general types of mergers and acquisitions: pure-cash and pure-share. The studies described above assume pure stock-for-stock (pure-share) mergers conditions in their models, with no cash-payment in the process. In stock-for-stock mergers, shareholders of both participating firm remain shareholders in the continuing combined enterprise. They negotiate the post-merger structure based on the pre-transaction ratio of their company market value. In the pure-cash type, the acquirer pays an equivalent cash amount to the target and receives all of the shares of the new combined firm. Shareholders of the acquiring firm are remained the only shareholders of the new company. However, Goergen and Renneboog [8] analyze 156 takeover bid samples in Europe during the 1990s; of the sample, 93 and 37 of 156 were pure-cash and pure-share, respectively, with 18 cases of the mixed cash-share type. In the mixed cash-share type, the acquirer pays both cash and shares to the target, and shareholders of both companies maintain ownership in the new company. A study by Faccio and Masulis [7] shows that 11.3% of a sample of European 3,667 mergers or acquisitions are mixed cash-share types, and these deals always have a larger transaction size. In Martynova and Renneboog [15], the proposition of the mixed cash-share bids increased to 19% in a sample of 1,361 European acquisitions.

This study develops a model of mixed cash-share mergers in which both the bidder and target remain shareholders of the new combined enterprise and negotiate over the post-merger sharing-rule. The bidder pays a cash premium to the target. Acquirers have a lot of motivations to pay a cash premium. In this paper, we assume that the bidder pays a cash premium to negotiate a higher post-merger share in the new company. This study aims to establish a joint merger model of timing, acquisition premium, and terms in markets with both perfect and imperfect information. Furthermore, the model will examine the effect of information on the decision-making.

In this study, we base our analysis on Morellec and Zhdanov [18]'s model of a joint determination of timing and terms of takeovers under competition and imperfect information. We extend the model to study the returns generated from asymmetric information. This study also differs from previous studies in several important dimensions. First, we consider a cash premium, which Morellec and Zhdanov [18] do not include. We establish the model using a non-cooperative game in which the bidder provides a tender offer, and the target can accept the offer or wait. The study also closely relates to Lukas and Welling [13], who develop a two-stage model analyzing the pricing and timing of mergers and acquisitions.

Second, we analyze the terms, assuming that the bidder and target will negotiate the terms of the post-merger combined enterprise, which we solve via a Nash barging solution (Nash [19]). Several studies combine game theory and real option theory. Azevedo and Paxson [2] discuss the discrete-time and continuous-time frameworks of a standard real option game and review two decades of academic research on standard and non-standard real option games. Lukas, Reuer and Welling [14] use a game-theoretic option approach to model the value of contingent earn-outs, finding that the firm will tend to postpone the investment under larger transaction costs, greater uncertainty in cash flows, a longer earn-out period, and higher performance targets.

Finally, we assume that both the bidder and target will probably mis-estimate the price, of which the managers of both firms can take advantage. In Morellec and Zhdanov [18], asymmetric information occurs between the participating firms and investors. The paper analyzes the abnormal returns from the announcement. We extend the model by assuming asymmetric information between two participating firms.

This study examines two scenarios. The first is a single bidder case in which we develop a basic model of the target acquisition in a market with perfect information without competition. We then extend the model to a second scenario with asymmetric information. Both the bidder and target may be mis-estimated in the market, which means the market price may be under- or over-estimated. However, both participating firms have complete information about their own company. Therefore, they will generate an optimal strategy using the estimated parameters based on the available information. The merger and acquisition activities can increase information disclosure and push the market price towards the real value. Therefore, the model generates abnormal returns to both the bidder and target around the merger.

The paper is organized as follows. In section 2, we introduce the model's framework and provide the basic model with full information. We then extend the basic model to one with imperfect information in Section 2.3. In section 3, we analyze the impact of the information on the results in Section 2. In Section 4, we give several numerical examples. Finally, Section 5 concludes.

2 The model

2.1 Basic framework

We construct the framework based on Morellec and Zhdanov [18]'s model. Consider two firms: a bidder and a target, which operate in the same market. We assume capital stocks of K_X for the bidder and K_Y for the target. The stock market valuation of each firm, denoted by $S_X(t)$ and $S_Y(t)$, respectively, without takeover is

$$S_X(t) = K_X X(t),\ S_Y(t) = K_Y Y(t), \tag{1}$$

where $X(t)$ and $Y(t)$ denote the per-unit value of capital, which follows a geometric Brownian motion,

$$\begin{aligned} dX(t) &= \mu_X X(t)\,dt + \sigma_X X(t)\,dW_X, \\ dY(t) &= \mu_Y Y(t)\,dt + \sigma_Y Y(t)\,dW_Y, \end{aligned} \tag{2}$$

where drift μ_X, $\mu_Y > 0$, and diffusion σ_X, $\sigma_Y > 0$ are constant parameters. W_X and W_Y are standard Brownian motions. The correlation coefficient between W_X and W_Y is constant, represented as $\rho \in (-1, 1)$.

We assume that all participants are risk neural, and the risk-less interest rate is r, $r \geq \mu_i (i = X, Y)$. Suppose the bidder aims to hold a certain fraction of the post-merger company to gain at least management rights in the company. To achieve this purpose, the bidder has to pay an acquisition premium to the target. We use parameter λ to denote the acquisition premium meaning that the bidder will pay $\lambda S_Y(t)$ to the target, where $S_Y(t)$ is the market value of the acquired company before the acquisition, given by equation (1). The bidder will receive a $\xi(\xi > 0)$ fraction of the post-merger combined entity in return. Consequently, the target gets a cash payment of $\lambda S_Y(t)$ and receives $(1-\xi)$ as the friction of the new combined company. The parameter λ can even be negative, in which case, the target shareholders bargain a higher post-merger ownership than the friction they could without cash payment. The bidder asks the target to pay for the higher bargain share. There is also the need to pay the transaction costs, denoted as cS_Y, when merging to the two companies.

From the target's standpoint, the higher the sale price $\lambda S_Y(t)$ is, the higher the cash payment they will receive upon selling. On the other hand, the remaining part of ownership will decrease, and the target will therefore receive a lower fraction $(1 - \xi)$. From the bidder's perspective, if they provide a higher payment to the target, the cost will increase, though they will accordingly gain a higher fraction ξ of the new entity. Therefore, there is an optimal payment strategy and sharing-rule ξ for both the bidder and target.

The acquisition process proceeds in two steps. At time t_0, the bidder offers a certain λ to the target, which can accept or reject the offer. The target firm will not decide immediately upon receiving the offer, and they can postpone the decision. Accepting the offer leads to an immediate merger of the two firms. The target firm will receive a payment of $\lambda S_Y(t)$ from the bidder and a $(1 - \xi)$ friction of the new combined entity, and has to give up a claim of $S_Y(t)$ instead.

Following Morellec and Zhdanov [18] and Shleifer and Vishny [20], we assume a linear combination of pre-takeover values in terms of per-unit value of capital. Hence, the post-merger value of the firm is

$$S_M\big(X(t), Y(t)\big) = S_X(t) + S_Y(t) + G\big(X(t), Y(t)\big), \tag{3}$$

where $G\big(X(t), Y(t)\big)$ is net synergy gains generated from the merger, given by

$$G\big(X(t), Y(t)\big) = K_Y\Big(\alpha\big(X(t) - Y(t)\big) - cY(t)\Big), (\alpha, c) \in \mathbb{R}^2_{++}, \tag{4}$$

where α is the synergy parameter, which all participants can observe. c denotes the per-unit merger cost of the capital value of the target firm. This assumption indicates that the synergy will be positive only when the bidder outperforms the target, which has the same meaning as $X(t) > Y(t)$.

2.2 A perfect information model

In this section, we consider a scenario with perfect information and no competition in the process. As in the framework, there are two stages in the merger process. We use the Markov Perfect Nash Equilibrium to analyze the strategy, in which the bidder will provide an optimized cash payment of $\lambda S_Y(t)$ ($\lambda > 0$) at stage one. The target receives this offer in stage one with a given λ and will accept the offer at the threshold of τ that maximizes their profit. The bidder and target will bargain the terms according to the optimized price λ and the threshold.

The target firm reacts to receiving a price offer of λ in stage one. They hold the option of accepting the offer. Conditional on the offered price λ, the target will choose a threshold τ in stage two and accept the offer. The acquisition premium to accept the offer is $\lambda S_Y(\tau)$. Therefore, the target will give up their claim, worth $S_Y(\tau)$, and receive both the payment value $\lambda S_Y(\tau)$ and a fraction $(1-\xi)$ of the new combined firm, worth $(1-\xi)S_M\big(X(\tau), Y(\tau)\big)$. The optimization function of the reacting party (target firm) at stage two is

$$f^{sale}\big(X(t), Y(t)\big) = \max_{\tau} \mathbb{E}\Big\{e^{-r\tau}\Big[(1-\xi)S_M\big(X(\tau), Y(\tau)\big) + \lambda S_Y(\tau) - S_Y(\tau)\Big]\Big\}. \tag{5}$$

Lemma 1 (Optimal threshold for the target in stage two) *Based on the value-maximizing strategy, the target firm will accept the offer and merge with the bidder when the ratio of capital price, denoted by $R(t) = X(t)/Y(t)$, reaches the level*

$$R^*(\lambda, \xi) = \Big[\big(\alpha + c - 1\big) + \frac{1-\lambda}{1-\xi}\Big]\frac{\vartheta_1}{\vartheta_1 - 1}\frac{K_Y}{K_X + \alpha K_Y}. \tag{6}$$

The value of merger option for the target is

$$f^{sale}\big(R(t)\big) = \begin{cases} \dfrac{(1-\xi)(\alpha + c - 1) + (1-\lambda)}{\vartheta_1 - 1} K_Y Y(t)\Big(\dfrac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}, & R(t) < R^*(\lambda) \\ \Big[(1-\xi)(K_X + \alpha K_Y)R(\tau) + \big[(1-\xi)(1-\alpha-c) + (\lambda - 1)\big]K_Y\Big]Y(t), & R(t) \geq R^*(\lambda), \end{cases} \tag{7}$$

and the first passing time is

$$\tau^* = \inf\{t | R(t) = R^*(\lambda, \xi)\}. \tag{8}$$

$\vartheta_1 > 1$ is the positive root of the quadratic equations

$$(\frac{1}{2}\sigma_X^2 + \frac{1}{2}\sigma_Y^2 - \rho\sigma_X\sigma_Y)\vartheta(\vartheta - 1) + (\mu_X - \mu_Y)\vartheta - (r - \mu_Y) = 0. \tag{9}$$

(Appendix A provides the proofs)

The threshold (6) is a reaction function of the cash payment parameter λ and the bargain sharing-rule $(1-\xi)$. Holding $(1-\xi)$ constant, a higher λ will decrease R^* and accelerate the merger. The threshold also largely relies on the ratio of firm size, represented by K_X/K_Y. A higher ratio of firm size (K_X/K_Y) will also accelerate the merger.

At stage two, the bidder will give up their claims, worth $S_X(\tau^*)$, and also pay the payment value of $\lambda S_Y(\tau^*)$. In return, they will receive a fraction ξ of the new combined firm, worth $\xi S_M\big(X(\tau^*), Y(\tau^*)\big)$.

The bidder will choose an optimal λ to maximize their benefit in stage one. The optimization function for the bidder is

$$f^{MA}\big(X(t),Y(t)\big) = \max_{\lambda} \mathbb{E}\Big\{e^{-r\tau^*}\Big[\xi S_M\big(X(\tau^*),Y(\tau^*)\big) - \lambda S_Y(\tau^*) - S_X(\tau^*)\Big]\Big\} \tag{10}$$

Proposition 1 (Optimal tender offer in stage one) *Maximizing the payoff function (10) yields the optimal offered portion λ^* from the bidder, given as*

$$\lambda^*(\xi) = \big[(\alpha+c)(1-\xi)+\xi\big] + \frac{\vartheta_1(\alpha+c)(1-\xi)(K_X+\alpha K_Y)}{(1-\xi)K_X - \alpha(\vartheta_1+\xi-1)K_Y} \tag{11}$$

Substituting result (11) into (6) yields

$$R^*(\xi) = \Big(\frac{\vartheta_1^2}{\vartheta_1-1}\Big)\frac{(\alpha+c)K_Y}{\alpha(\vartheta_1+\xi-1)K_Y-(1-\xi)K_X} \tag{12}$$

Receiving the optimal offer of λ^*, the target firm will accept the offer when the ratio of capital price $R(t)$ satisfies (12). The bidder will immediately merge with the target after the target accepts the offer. (Appendix B contains the proofs)

According the result from (11), $d\lambda^*(\xi)/\,d(1-\xi) < 0$ (equivalently, $d\lambda^*(\xi)/\,d\xi > 0$), the cash payment parameter λ^* positively relates to the sharing-rule ξ. Hence, the bidder is willing to pay a higher cash premium to the target for a higher post-merger share ξ. On the other hand, a target's higher $(1-\xi)$ requirement will decrease λ^* and increase the R^*, slowing the merging process.

We suppose the bidder and target will negotiate the post-merger sharing-rule at stage one. As in Section 2.1, the post-merger sharing-rule is ξ for the bidder and $(1-\xi)$ for the target. We obtain the optimal ξ using the Nash bargaining solution. The optimization function is

$$\Pi\big(X(t),Y(t)\big) = \max_{\xi}\Big[f^{MA}\big(X(t),Y(t);\xi\big)\Big]^{\beta}\Big[f^{sale}\big(X(t),Y(t);\xi\big)\Big]^{1-\beta}, \tag{13}$$

where the bargain power parameter β is subject to $\beta \in (0,1)$. Solving the maximization problem (13) yields the following.

Proposition 2 (Optimal merger terms) *The bidder will pay the optimal offered portion λ^* and expect to receive the optimal ξ^* fraction of the combined entity, where ξ^* satisfies*

$$\xi^* = 1 - \frac{(1-\beta)\alpha K_Y}{K_X+\alpha K_Y}. \tag{14}$$

Receiving the optimal offered portion λ^*, the optimal strategy for the target is to require $(1-\xi^*)$ as a fraction of the combined entity and choose to accept the offer at τ^*. The terms for the target are

$$1-\xi^* = \frac{(1-\beta)\alpha K_Y}{K_X+\alpha K_Y}. \tag{15}$$

(Appendix C provides the proofs)

The optimal sharing-rule (equation (14)) under a Nash bargaining solution positively relates to the ratio of the firm size (K_X/K_Y). With a higher firm size (K_X/K_Y), the bidder is willing to require a higher ξ^* and therefore pay a higher cash premium.

2.3 An imperfect information model

In this section, we analyze the strategy in a market with asymmetric information. In this scenario, it is possible for the market price to be misjudged. In contrast to assumption (4), the net synergy gains generated from the merger here are

$$G\big(X(t),Y(t)\big) = K_Y\Big(\alpha\big(\omega_B X(t) - \omega_T Y(t)\big) - cY(t)\Big), (\alpha, c) \in \mathbb{R}^2_{++}, \tag{16}$$

where $\omega_B > 0$ and $\omega_T > 0$ are information parameters. The market price is underestimated if $\omega_i \in (1,\infty), (i = B, T)$, and vice versa. We assume that ω_B is only observable to the managers of the bidder and ω_T is only observable to the managers of the target. Thus, managers know only the real value of their own company. Additionally, managers cannot trade on their inside information due to legal prohibitions.

For the bidder, the net synergy gains generated from the merger are

$$G^B\big(X(t),Y(t)\big) = K_Y\Big(\alpha\big(\omega_B X(t) - Y(t)\big) - cY(t)\Big); \tag{17}$$

ω_T is not observable to the bidder, who believes that the target firm's market price is the real price. Based on the bidder's information, the post-merger value of the combined firm should be

$$S^B_M\big(X(t),Y(t)\big) = S_X(t) + S_Y(t) + G^B\big(X(t),Y(t)\big). \tag{18}$$

For the target, the net synergy gains generated from the merger are

$$G^T\big(X(t),Y(t)\big) = K_Y\Big(\alpha\big(X(t) - \omega_T Y(t)\big) - cY(t)\Big), \tag{19}$$

Based on the target firm's information, the post-merger value of the combined firm should be

$$S^T_M\big(X(t),Y(t)\big) = S_X(t) + S_Y(t) + G^T\big(X(t),Y(t)\big). \tag{20}$$

Asymmetric information will be disclosed after the merger. Therefore, the merger of the two firms will generate an abnormal return.

Similar to the model in the perfect information market, the optimization function of the reacting party (target firm) at stage two is

$$\tilde{f}^{sale}\big(X(t),Y(t)\big) = \max_{\tau} \mathbb{E}\Big\{e^{-r\tau}\Big[(1-\xi)S^T_M\big(X(\tau),Y(\tau)\big) + \lambda S_Y(\tau) - S_Y(\tau)\Big]\Big\}. \tag{21}$$

Lemma 2 (Optimal threshold in stage two in a market with imperfect information) *Based on the value-maximizing strategy, the target firm will accept the offer and merge with the bidder when the capital price ratio $R(t)$ reaches*

$$\tilde{R}^*(\lambda,\xi) = \Big[\big(\alpha\omega_T + c - 1\big) + \frac{1-\lambda}{1-\xi}\Big]\frac{\vartheta_1}{\vartheta_1 - 1}\frac{K_Y}{K_X + \alpha K_Y}. \tag{22}$$

The value of merger option for the target is

$$\tilde{f}^{sale}\big(R(t)\big) = \begin{cases} \dfrac{(1-\xi)(\alpha\omega_T - c - 1) + (1-\lambda)}{\vartheta_1 - 1} K_Y Y(t)\Big(\dfrac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}, & R(t) < R^*(\lambda) \\ \Big[(1-\xi)(K_X + \alpha K_Y)R(t) + \big[(1-\xi)(1-\alpha\omega_T - c) + (\lambda - 1)\big]K_Y\Big]Y(t), & R(t) \geq R^*(\lambda) \end{cases}$$

(23)

The first passing time is

$$\tilde{\tau}^* = \inf\{t|R(t) = \tilde{R}^*(\lambda, \xi)\}, \tag{24}$$

where $\vartheta_1 > 1$ is the positive root of equation (9). (Appendix D provides the proofs)

At stage one, the bidder's optimization function is

$$\tilde{f}^{MA}\big(X(t), Y(t)\big) = \max_{\tilde{\lambda}} \mathbb{E}\Big\{e^{-r\tau^*}\Big[\xi S_M^B\big(X(\tau^*), Y(\tau^*)\big) - \tilde{\lambda} S_Y(\tau^*) - S_X(\tau^*)\Big]\Big\}. \tag{25}$$

With the available information, the bidder will estimate the target firm's threshold as $R^*(\tilde{\lambda})$, which is given by equation (6).

Proposition 3 (Optimal tender offer in stage one in a market with imperfect information) *The information parameter ω_T is not observable before $\tilde{\tau}$; the bidder then optimizes the acquisition premium under a distorted estimation. The optimal offered portion $\tilde{\lambda}^*$ the bidder chooses is*

$$\tilde{\lambda}^*(\xi) = (1-\xi)\big(\alpha + c\big) + \xi + \frac{\vartheta_1(\alpha + c)(1-\xi)(K_X + \alpha K_Y)}{(1-\xi)K_X - \alpha[(\vartheta_1 + \xi - 1) + \vartheta_1\xi(\omega_B - 1)]K_Y}. \tag{26}$$

Given $\tilde{\lambda}^*(\xi)$, the threshold finally is equal to $\tilde{R}^*(\tilde{\lambda}^*, \xi)$. According to the result of (22), the optimal threshold to accept the offer is

$$\tilde{R}^*(\xi) = \frac{\vartheta_1}{\vartheta_1 - 1}\Big[\frac{\alpha(\omega_T - 1)K_Y}{K_X + \alpha K_Y} + \frac{\vartheta_1(\alpha + c)K_Y}{\alpha[(\vartheta_1 + \xi - 1) + \vartheta_1\xi(\omega_B - 1)]K_Y - (1-\xi)K_X}\Big] \tag{27}$$

(Appendix E provides the proofs)

Similar to the model in the perfect information market, the optimization function of the sharing-rule is

$$\tilde{\Pi}\big(X(t), Y(t)\big) = \max_{\tilde{\xi}} \Big[\tilde{f}^{MA}\big(X(t), Y(t); \tilde{\xi}\big)\Big]^{\beta}\Big[\tilde{f}^{sale}\big(X(t), Y(t); \tilde{\xi}\big)\Big]^{1-\beta} \tag{28}$$

where the bargaining power parameter β is subject to $\beta \in (0, 1)$. Solving the maximization problem (28) yields the following result.

Proposition 4 (Optimal merger terms in a market with imperfect information) *The bidder will pay the optimal offered portion $\tilde{\lambda}^*$ and expect to receive the optimal $\tilde{\xi}^*$ fraction of the combined entity. Receiving the optimal offered portion $\tilde{\lambda}^*$, the optimal strategy for the target is to require $(1-\tilde{\xi}^*)$ as a fraction of the combined entity and choose to accept the offer at $\tilde{\tau}^*$. The optimal merger terms in a market with imperfect information satisfies*

$$\frac{\alpha}{\alpha + c}\frac{K_Y}{K_X + \alpha K_Y}\frac{\omega_B}{1 - \tilde{\xi}^*} = \frac{\gamma_1}{\gamma_2^2} + \frac{(1-\beta)\gamma_1}{\beta\vartheta_1\gamma_2^2 - (1-\beta)(1-\vartheta_1)\gamma_1\gamma_2} \tag{29}$$

where

$$\gamma_1 = (1-\tilde{\xi}^*)(\alpha + c - 1) + (1 - \tilde{\lambda}^*(\tilde{\xi}^*)) \tag{30}$$

$$\gamma_2 = (1-\tilde{\xi}^*)(\alpha\omega_T + c - 1) + (1 - \tilde{\lambda}^*(\tilde{\xi}^*)) \tag{31}$$

(Appendix F provides the proofs.)

The entire strategy will be decided in stage one; therefore, both participating firms have no chance to adjust the strategy after the decision. The optimal sharing-rule under a Nash bargaining solution in the imperfect information market is the solution to equation (29). For the bidder, the strategy will be $(\tilde{\lambda}^*(\tilde{\xi}^*),\ \tilde{\xi}^*)$. For the target, the strategy will be $(\tilde{R}^*(\tilde{\lambda},\tilde{\xi}^*),\ 1-\tilde{\xi}^*)$. In the next section, we analyze the impact of the information on the strategy.

3 Impact of information

All information is revealed after the entire merging process is complete. After merging, the market price reflects the real value of the enterprise. Before merging, the participating firms know only their own real value and estimate their counterpart's value assuming that their value is equal to their market value. Because the information parameter ω_T is unknown to the bidder until after the merger, ω_T has no impact on the decision, which is $\tilde{\lambda}^*$ for the bidder. Comparing the results of (11) and (26), the higher the information parameter ω_B is, the higher the acquisition premium the bidder will provide. In the range $\omega_B \in (0,1)$, $\tilde{\lambda}^* < \lambda^*$ and the reversal relationship is in the range $\omega_B \in (1,\infty)$.

Proposition 5 (Impact of information on the acquisition premium) *The bidder is willing to pay a higher acquisition premium when their firm is underestimated, and vice versa. A higher acquisition premium therefore accelerates the merger process and the disclosure of underestimated information.*

The target will choose their strategy following the reaction function (22), the effect of information on their function is as follows.

Proposition 6 (Impact of information on the threshold) *If the parameters satisfy*

$$(1-\xi)\alpha(\omega_T-1) > \tilde{\lambda}^* - \lambda^*, \tag{32}$$

or equivalently,

$$\frac{\omega_T-1}{\omega_B-1} > \frac{\xi\vartheta_1^2(\alpha+c)(K_X+\alpha K_Y)K_Y}{[(1-\xi)K_X-\alpha(\vartheta_1+\xi-1)K_Y)][(1-\xi)K_X-\alpha[(\vartheta_1+\xi-1)+\vartheta_1\xi(\omega_B-1)K_Y]}, \tag{33}$$

the threshold in the situation with imperfect information is higher than that with perfect information, which leads to

$$\tilde{R}^* > R^*, \tag{34}$$

and vice versa.

If $\omega_B \in (0,1)$ and $\omega_T \in (1,\infty)$, the left side of (32) is positive and the right side is negative. The inequality (32) will always be satisfied. If $\omega_B \in (1,\infty)$ and $\omega_T \in (0,1)$, the inequality (32) will never be satisfied because of the different signs of the both sides. Therefore, if the target is overvalued and the bidder is undervalued, inequality (32) will never be satisfied, and vice versa. If inequality (32) cannot be satisfied, the condition $\tilde{R}^* < R^*$ always holds. In this situation, the symmetric information will always accelerate the merger. On the other hand, if both of the target and bidder are overestimated, which leads to $\omega_T \in (0,1)$ and $\omega_B \in (0,1)$, inequality (33) means that the higher the target is overestimated, the higher the threshold is. Additionally, if both of the target and bidder are underestimated, which leads to $\omega_T \in (1,\infty)$ and $\omega_B \in (1,\infty)$, inequality (33) means that the higher the target is underestimated, the higher the threshold is. Therefore, if the mis-estimations of per-unit market value of capital are in the same direction, the higher the distortion is and the slower the merger process will be.

When the bidder and target merge, they will receive a certain fraction of the post-merging firm under the real net synergy gain from merging, given by (16). We can write the value of the bidder pre-merger as

$$S_X(\tilde{\tau}^*_-) = K_X X(\tilde{\tau}^*_-) + \tilde{f}^{MA}\big(X(\tilde{\tau}^*_-), Y(\tilde{\tau}^*_-); (\omega_B, 1)\big). \tag{35}$$

In addition, the value of the target pre-merger is

$$S_Y(\tilde{\tau}^*_-) = K_Y Y(\tilde{\tau}^*_-) + \tilde{f}^{sale}\big(X(\tilde{\tau}^*_-), Y(\tilde{\tau}^*_-); (1, \omega_T)\big). \tag{36}$$

The abnormal returns satisfy

$$\begin{aligned} AR_X(\tilde{\tau}^*) =& \frac{\tilde{f}^{MA}\big(X(\tilde{\tau}^*), Y(\tilde{\tau}^*); (\omega_B, \omega_T)\big) - \tilde{f}^{MA}\big(X(\tilde{\tau}^*), Y(\tilde{\tau}^*); (\omega_B, 1)\big)}{S_X(\tilde{\tau}^*_-)}; \\ =& \frac{\xi\alpha(1-\omega_T)K_Y}{\xi(K_X + \alpha\omega_B K_Y)\tilde{R}^* + [\xi(1-\alpha-c) - \tilde{\lambda}^*]K_Y}. \end{aligned} \tag{37}$$

$$\begin{aligned} AR_Y(\tilde{\tau}^*) =& \frac{\tilde{f}^{sale}\big(X(\tilde{\tau}^*), Y(\tilde{\tau}^*); (\omega_B, \omega_T)\big) - \tilde{f}^{sale}\big(X(\tilde{\tau}^*), Y(\tilde{\tau}^*); (1, \omega_T)\big)}{S_Y(\tilde{\tau}^*_-)}; \\ =& \frac{(1-\xi)\alpha(\omega_B - 1)K_Y R^*}{(1-\xi)(K_X + \alpha K_Y)\tilde{R}^* + [(1-\xi)(1-\omega_T\alpha - c) + \tilde{\lambda}^*]K_Y}. \end{aligned} \tag{38}$$

In the next section, we examine the abnormal returns and compare the results with previous studies.

4 Numerical examples

This section provides several numerical tests of the results. Hackbarth and Morellec [10] find that acquiring firms earn low or negative abnormal returns, while target firms earn substantially positive abnormal returns around the announcement date of the takeover. We first study the abnormal returns in this model and compare the results with those reported in the literature. Second, we examine the acquisition premium and the reaction threshold of merger changes with endogenous parameters. Third, we provide insights into effect of information on the results. Table 1 summarizes the basic parameter values.

Table 1: Parameter values.

Parameter		Value
Drift of the bidder	μ_X	0.05
Drift of the target	μ_Y	0.02
Volatility of the bidder	σ_X	0.20
Volatility of the target	σ_Y	0.30
Ratio of the firm size	K_X/K_Y	3.00
Correlation coefficient	ρ	0.50
Synergy parameter	α	0.30
Per unit merger cost	c	0.01
Risk-free rate	r	0.06

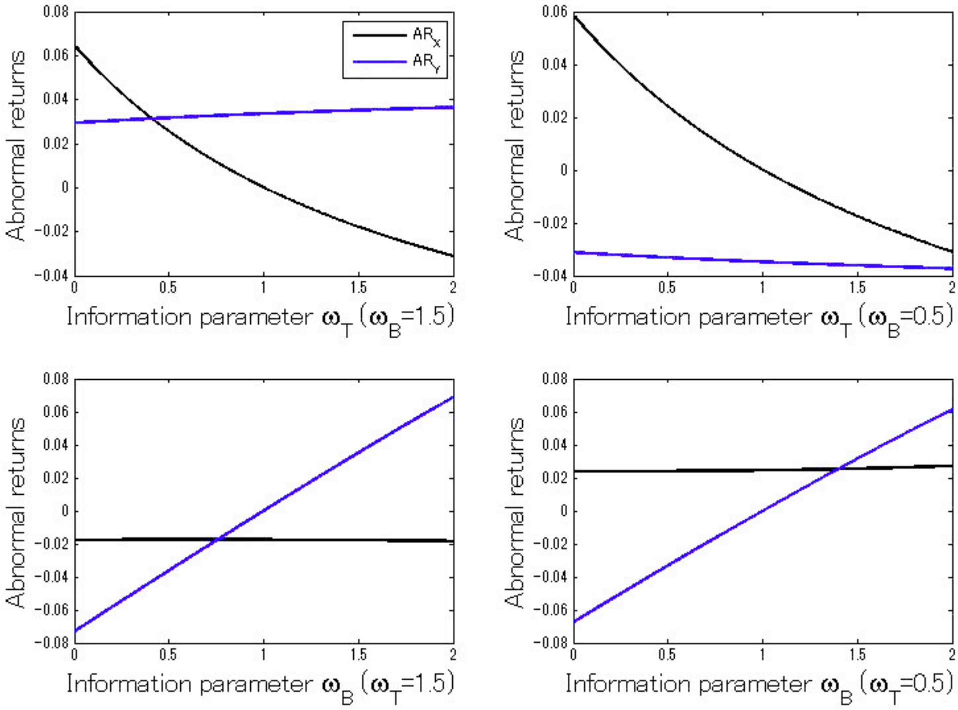

Figure 1: The effects of information on abnormal returns. The black and blue lines represent the bidder and target's abnormal returns, respectively. We first test the abnormal return change with the information parameter ω_T, assuming $\omega_B = 1.5$ and $\omega_B = 0.5$, meaning that the bidder is under- or over-estimated. We then test the abnormal return change with the information parameter ω_B, assuming $\omega_T = 1.5$ and $\omega_T = 0.5$.

Figure 1 plots the abnormal returns as a function of the information parameters ω_B and ω_T. In the figures, the participating firms' abnormal returns can be positive or negative according to the relationship between ω_B and ω_T. Figure 1 shows a different result than that reported in Hackbarth and Morellec [10], in which the abnormal returns to the target shareholders are not always higher than those of the bidder are. The abnormal return to the target shareholders is largely influenced by

ω_B and increases with ω_B. The more the bidder is underestimated, the more the abnormal return to the target shareholders is. The zero abnormal return point to the target shareholders is around $\omega_B = 1$. The impact of ω_T was observable prior to the merger, and hence has less impact on the target's abnormal return. The abnormal return to the bidder shareholders is largely influenced by ω_T and decrease with ω_T. For the bidder, ω_B was observable before the merger and thus has less influence.

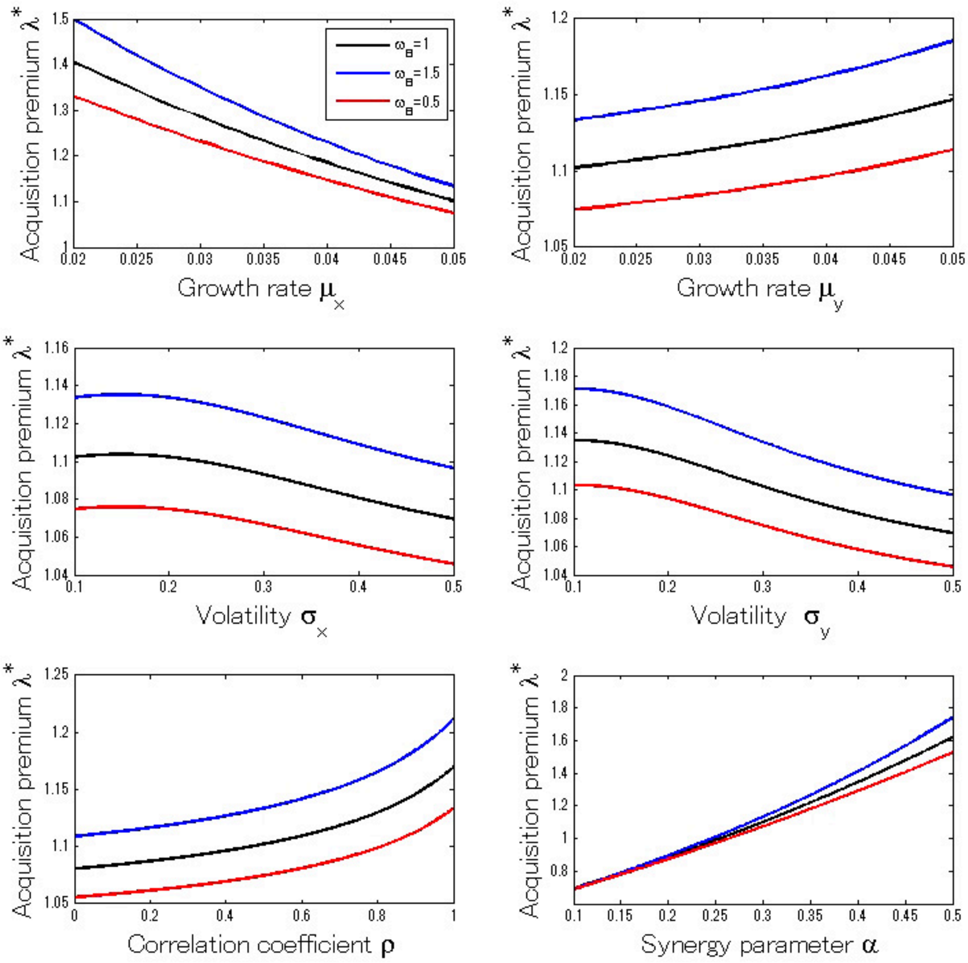

Figure 2: Parameter impacts on the acquisition premium λ^* under $\xi = 0.5$. We assume that the bidder and target will negotiate a bisection sharing-rule and test the acquisition premium with the growth rate, volatility, correlation coefficient, and synergy parameter. The black, red, and blue lines represent the relationship when $\omega_B = 1$, $\omega_B = 0.5$, and $\omega_B = 1.5$, respectively. ω_T does not affect the bidder's strategy. Therefore, we set $\omega_T = 1$.

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The merger's timing and acquisition premium depend on the growth rate and volatility of the firms' core business valuations. We release the basic parameter values, which we want to test, and

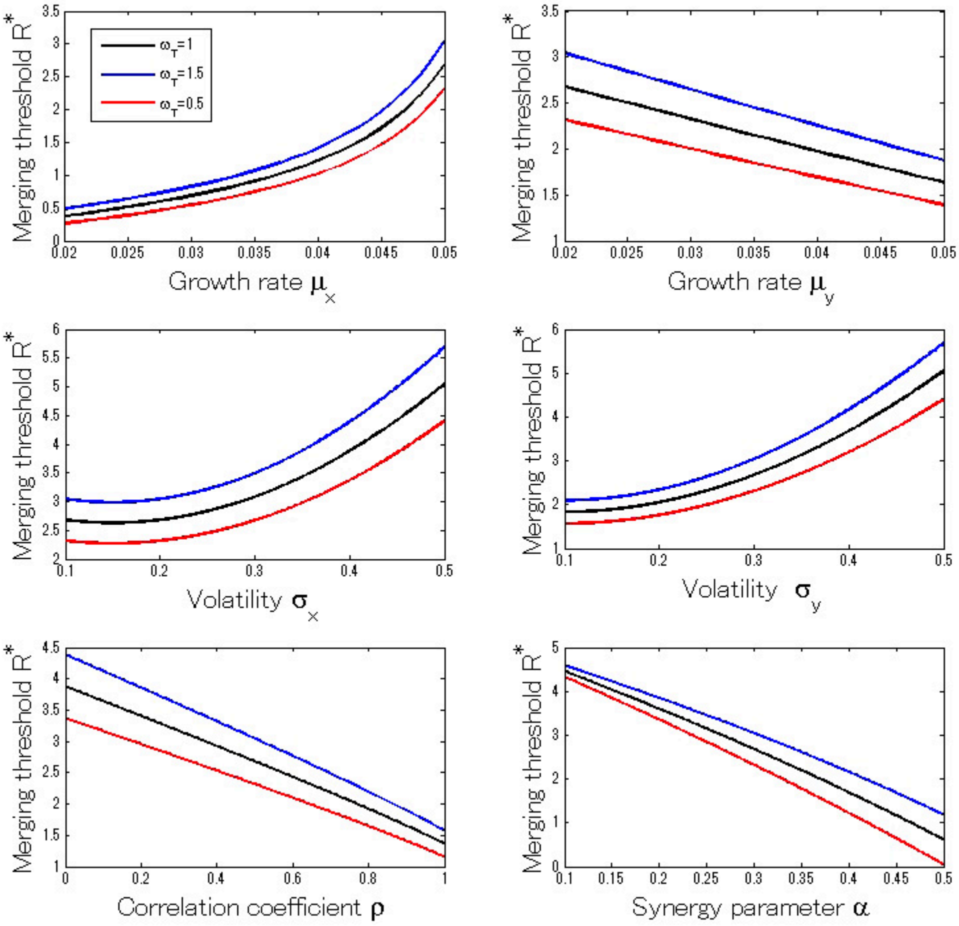

Figure 3: Parameter impacts on the threshold R^* under $\xi = 0.5$. We again assume that the bidder and target will negotiate a bisection sharing-rule and test the acquisition premium with the growth rate, volatility, correlation coefficient, and synergy parameter. The black, red, and blue lines represent the relationship when $\omega_T = 1$, $\omega_T = 0.5$, and $\omega_T = 1.5$, respectively. ω_B does not affect the bidder's strategy. Therefore, we set $\omega_B = 1$.

fix the others, as in Table 1. In Figure 2, the bidder will pay a lower acquisition premium when the bidder's growth rate is high, and pay more when the target's growth rate is high. As in the reaction function (22), the lower the acquisition premium is, the higher the threshold will be. Therefore, in Figure 3, a bidder's higher growth rate will decelerate the merger process, and a target's higher growth rate will accelerate the process. The parameter ϑ_1 has a positive relationship with volatility σ_X when $\sigma_X < \rho\sigma_Y$, and vice versa. Thus, the acquisition premium will increase with volatility σ_X and then decrease. When the volatility both firms is high, the bidder will pay a lower acquisition premium. Hence, it represent a reverse relationship in Figure 3. The correlation coefficient and synergy parameter will also increase the acquisition premium. In Figure 2, information ω_B increases the acquisition premium. The blue line is always above the other two lines in both Figure 2 and Figure 3.

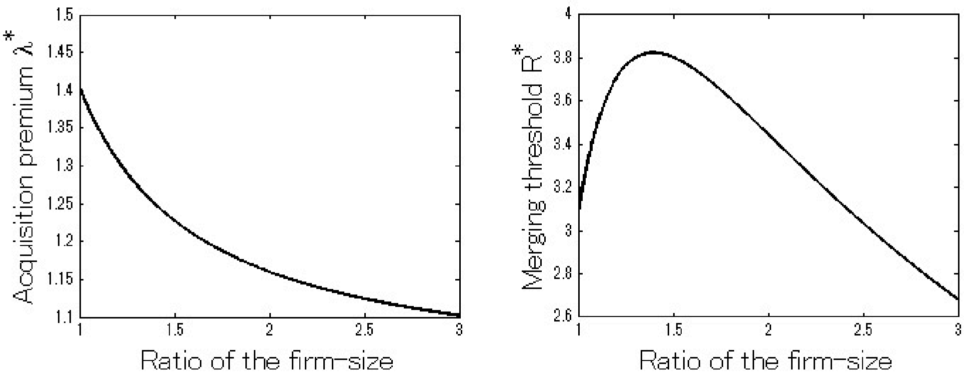

Figure 4: Effect of the firm size on strategy decision. Firm size equals K_X/K_Y. We assume $\omega_T = 1$ and $\omega_B = 1$ in this figure and test the impacts of firm size of the bidder and target. Hence, the figure represents the relationship in a prefect information market.

The ratio of the two participating firms always decreases the acquisition premium. A small target will generate a small synergy according to the assumption in (4). Therefore, the bidder will pay a lower acquisition premium. Although the threshold R^* is a reaction function of the acquisition premium λ^*, a lower λ^* will increase the threshold and decelerate the process. On the other hand, the ratio K_X/K_Y has a direct negative effect on the threshold if given a fixed λ^*. The target has two source of returns: the cash premium payment $\lambda S_Y(\tau_-^*)$, which decreases with K_X/K_Y; and the share payment $(1-\xi)$. When K_X/K_Y is large, the target shareholders will gain a relatively high post-merger share payment (the $(1-\xi)K_X X(\tau_-^*)$ part is higher), even under lower acquisition premium. And therefore accelerates the merging process. Thus, the threshold will have a maximum when the merger process is slowest.

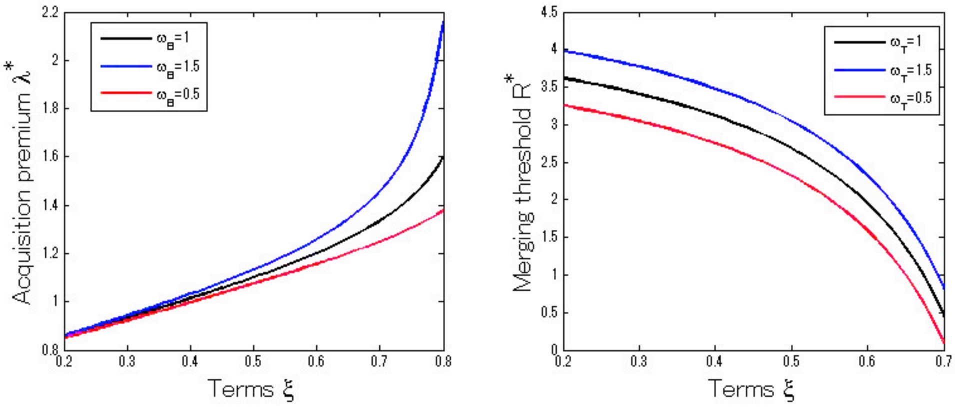

Figure 5: Relationship between the terms ξ, acquisition premium, and merger threshold. The left side represents the bidder's strategy and we set $\omega_T = 1$. The left side represents the target's strategy. Therefore, $\omega_B = 1$.

In Figure 5, if the bidder negotiates a higher post-merger share ξ, they need to pay a higher acquisition premium. The bidder may expect more management control after the merger and will provide the target with an addition cash payment. The target firm is willing to receive a higher cash payment; therefore, the merger threshold is also lower when ξ is high. In terms of the information parameter, an underestimated bidder will increase the acquisition premium, and an underestimated target decelerates the merger process.

5 Conclusion

This paper develops a dynamic model of a joint takeover to determine the timing, acquisition premiums, and terms of a takeover. The model considers a mixed cash-share payment, with both acquisition premiums and a sharing-rule between the bidder and target. We extend the model to a market with imperfect information and show how the strategy interacts with asymmetric information. The merger generates an abnormal return. The model also tests the relationship between the abnormal return and information parameters.

The timing, acquisition premiums, and terms of the takeover interact. The model predicts that (i) the abnormal returns to the bidder's shareholders are positive when the target is overestimated and negative when the target is underestimated, (ii) the abnormal returns to the target's shareholders are negative when the bidder is overestimated and positive when the bidder is underestimated, (iii) the abnormal return to the target's shareholders can be higher or lower than those to the bidder's shareholders, (iv) a higher acquisition premium will accelerate the merger process, and (v) an undervalued bidder will accelerate the merger process and an underestimated target will decelerate the merger process.

A Proof of Lemma 1

Substituting the firm's post-merger value given by equation (3) into the value of the option for the target to accept the offer, given by (5), yields

$$f^{sale}\big(X(t),Y(t)\big) = \max_{\tau} \mathbb{E}\Big\{e^{-r\tau}\Big[(1-\xi)\Big(S_X(\tau)+S_Y(\tau)+G_M\big(X(\tau),Y(\tau)\big)\Big)+(\lambda-1)K_Y Y(\tau)\Big]\Big\} \tag{A.1}$$

which is in the region for the two state variables. The payoff function (A.1) satisfies the partial differential equation

$$rf^{sale}\big(X(t),Y(t)\big) = \mu_X X f_X^{sale} + \mu_Y Y f_Y^{sale} + \frac{1}{2}\sigma_X^2 X^2 f_{XX}^{sale} + \frac{1}{2}\sigma_Y^2 Y^2 f_{YY}^{sale} + \rho\sigma_X\sigma_Y XY f_{XY}^{sale}. \tag{A.2}$$

The value function (A.1) is linearly homogeneous in $X(t)$ and $Y(t)$. We set the ratio of the two capital prices as

$$R(t) = \frac{X(t)}{Y(t)} \tag{A.3}$$

We can rewrite the synergy equation (4) as

$$G_M\big(X(t),Y(t)\big) = Y(t)g_R^G\big(R(t)\big), \tag{A.4}$$

where

$$g_R^G\big(R(t)\big) = K_Y\big[\alpha\big(R(t)-1\big)-c\big]. \tag{A.5}$$

Substituting equations (A.4) and (A.5) into (A.1) yields the payoff function

$$f^{sale}\big(X(t),Y(t)\big) = \max_{\tau} \mathbb{E}\Big\{e^{-r\tau}Y(\tau)\Big[(1-\xi)(K_X+\alpha K_Y)R(\tau)+\big[(1-\xi)(1-\alpha-c)+(\lambda-1)\big]K_Y\Big]\Big\} \tag{A.6}$$

The payoff function above increases with the ratio $R(t)$. We can assume that when the ratio reaches a certain threshold R^*, the target firm will obtain the offer and merge with the bidder. Suppose

$$f^{sale}\big(X(t),Y(t)\big) = Y(t)g^{sale}\big(R(t)\big). \tag{A.7}$$

Successive differentiation equations of (A.7) with respect to $R(t)$ give

$$f_X^{sale}\big(X(t),Y(t)\big) = g_R^{sale}\big(R(t)\big), \tag{A.8}$$
$$f_Y^{sale}\big(X(t),Y(t)\big) = g^{sale}\big(R(t)\big) - R(t)g_R^{sale}\big(R(t)\big), \tag{A.9}$$
$$f_{XX}^{sale}\big(X(t),Y(t)\big) = g_{RR}^{sale}\big(R(t)\big)/Y(t), \tag{A.10}$$
$$f_{YY}^{sale}\big(X(t),Y(t)\big) = R(t)^2 g_{RR}^{sale}\big(R(t)\big)/Y(t), \tag{A.11}$$
$$f_{XY}^{sale}\big(X(t),Y(t)\big) = -R(t)g_{RR}^{sale}\big(R(t)\big)/Y(t). \tag{A.12}$$

Substituting equations (A.9) - (A.12) into (A.2) yields the ordinary differential equation

$$(\frac{1}{2}\sigma_X^2+\frac{1}{2}\sigma_Y^2-\rho\sigma_X\sigma_Y)R^2 g_{RR}^{sale}\big(R(t)\big) + (\mu_X-\mu_Y)Rg_R^{sale}\big(R(t)\big) - (r-\mu_Y)g^{sale}\big(R(t)\big) = 0. \tag{A.13}$$

Using the technique for solving O.D.E., the general solution of (A.13) can be written as $g^{sale}\big(R(t)\big) = AR(t)^{\vartheta_1} + CR(t)^{\vartheta_2}$, where $\vartheta_1 > 1$ and $\vartheta_2 < 0$ are roots of the quadratic equation

$$(\frac{1}{2}\sigma_X^2 + \frac{1}{2}\sigma_Y^2 - \rho\sigma_X\sigma_Y)\vartheta(\vartheta-1) + (\mu_X - \mu_Y)\vartheta - (r - \mu_Y) = 0. \tag{A.14}$$

As the no-bubble condition $\lim\limits_{R(t)\to 0} g^{sale}\big(R(t)\big) = 0$, we have the solution

$$g^{sale}\big(R(t)\big) = AR(t)^{\vartheta_1}, \vartheta_1 > 1. \tag{A.15}$$

We can solve $g^{sale}\big(R(t)\big)$ subject to the value-matching and smoothing-pasting conditions

$$\begin{aligned} &g^{sale}\big(R(t)\big)|_{t=\tau^*} = g^{G}\big(R(t)\big)|_{t=\tau^*}, \\ &g_R^{sale}\big(R(t)\big)|_{t=\tau^*} = g_R^{G}\big(R(t)\big)|_{t=\tau^*}. \end{aligned} \tag{A.16}$$

The target will accept the offer at τ^*, which leads to $R(t) = R^*$ at τ^*. Substituting equations (A.6) and (A.15) into (A.16) yields

$$\begin{aligned} &A(R^*)^{\vartheta_1} = (1-\xi)(K_X + \alpha K_Y)R^* + \big[(1-\xi)(1-\alpha-c) + (\lambda - 1)\big]K_Y, \\ &\vartheta_1 A(R^*)^{\vartheta_1 - 1} = (1-\xi)(K_X + \alpha K_Y). \end{aligned} \tag{A.17}$$

Solving the equations above yields

$$R^*(\lambda) = \Big[\big(\alpha + c - 1\big) + \frac{1-\lambda}{1-\xi}\Big]\frac{\vartheta_1}{\vartheta_1 - 1}\frac{K_Y}{K_X + \alpha K_Y}, \tag{A.18}$$

$$A = \frac{(1-\xi)(\alpha+c-1) + (1-\lambda)}{\vartheta_1 - 1}K_Y(R^*(\lambda))^{-\vartheta_1} \tag{A.19}$$

Substituting the results above into (A.15) yields

$$g^{sale}\big(R(t)\big) = \frac{(1-\xi)(\alpha+c-1) + (1-\lambda)}{\vartheta_1 - 1}K_Y\Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}. \tag{A.20}$$

Therefore,

$$f^{sale}\big(X(t), Y(t)\big) = Y(t)\frac{(1-\xi)(\alpha+c-1) + (1-\lambda)}{\vartheta_1 - 1}K_Y\Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}, \tag{A.21}$$

where $R(t) = X(t)/Y(t)$.

B Proof of proposition 1

Submitting the result of Lemma 1 into the optimization function (10), we have

$$f^{MA}\big(X(t), Y(t)\big) = \max_{\lambda} Y(t)\Big\{\Big[\big((\xi-1)K_X + \xi\alpha K_Y\big)R^*(\lambda) + \big(\xi(1-\alpha-c) - \lambda\big)K_Y\Big]\Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}\Big\} \tag{B.1}$$

The value function (B.1) is linearly homogeneous in $X(t)$ and $Y(t)$; thus, we assume

$$f^{MA}\big(X(t), Y(t)\big) = Y(t)g^{MA}\big(R(t)\big), \tag{B.2}$$

where

$$g^{MA}\big(R(t)\big) = \max_{\lambda}\Big\{\Big[\big((\xi-1)K_X + \xi\alpha K_Y\big)R^*(\lambda) + \big(\xi(1-\alpha-c)-\lambda\big)K_Y\Big]\Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}\Big\} \tag{B.3}$$

We can solve the optimization problem when

$$\frac{dg^{MA}\big(R(t)\big)}{d\lambda} = 0 \tag{B.4}$$

The result of condition (B.4) is

$$\lambda^* = (\alpha+c)(1-\xi) + \xi + \frac{\vartheta_1(\alpha+c)(1-\xi)(K_X+\alpha K_Y)}{(1-\xi)K_X - \alpha(\vartheta_1+\xi-1)K_Y} \tag{B.5}$$

Therefore,

$$R^* = \Big(\frac{\vartheta_1^2}{\vartheta_1-1}\Big)\frac{(\alpha+c)K_Y}{\alpha(\vartheta_1+\xi-1)K_Y - (1-\xi)K_X} \tag{B.6}$$

C Proof of proposition 2

We can solve the maximization problem (13) when the sharing-rule satisfies

$$\beta\frac{df^{MA}\big(X(t),Y(t);\xi\big)}{d\xi} + (1-\beta)\frac{f^{MA}\big(X(t),Y(t);\xi\big)}{f^{sale}\big(X(t),Y(t);\xi\big)}\frac{df^{sale}\big(X(t),Y(t);\xi\big)}{d\xi} = 0 \tag{C.1}$$

According to equations (A.7) and (B.2), we can rewrite equation (C.1) as

$$\beta\frac{dg^{MA}\big(R(t);\xi\big)}{d\xi} + (1-\beta)\frac{g^{MA}\big(R(t);\xi\big)}{g^{sale}\big(R(t);\xi\big)}\frac{dg^{sale}\big(R(t);\xi\big)}{d\xi} = 0 \tag{C.2}$$

Substituting the optimal acquisition premium λ^*, given by (11), and the threshold (12) into the payoff function (B.3), yields

$$g^{MA}\big(R(t);\xi\big) = \frac{(\alpha+c)K_Y}{\vartheta_1-1}\Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1} \tag{C.3}$$

The first-order differentiation of equation (C.3) with respect to ξ is

$$\frac{dg^{MA}\big(R(t);\xi\big)}{d\xi} = \frac{\vartheta_1}{\vartheta_1-1}\frac{(\alpha+c)K_Y(\alpha K_Y+K_X)}{\alpha(\vartheta_1+\xi-1)K_Y-(1-\xi)K_X}\Big(\frac{R(t)}{R^*(\xi)}\Big)^{\vartheta_1} \tag{C.4}$$

We simplify the payoff function of (A.20) as

$$g^{sale}\big(R(t);\xi\big) = \frac{\vartheta_1}{\vartheta_1-1}\frac{(1-\xi)(\alpha+c)(K_X+\alpha K_Y)K_Y}{\alpha\vartheta_1K_Y-(1-\xi)(K_X+\alpha K_Y)}\Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1} \tag{C.5}$$

The first-order differentiation of equation (C.5) with respect to ξ is

$$\frac{dg^{sale}\big(R(t);\xi\big)}{d\xi} = \Big(\frac{R(t)}{R^*(\lambda)}\Big)^{\vartheta_1}\frac{\vartheta_1^2}{\vartheta_1-1}\frac{(\alpha+c)\big((1-\xi)(K_X+\alpha K_Y)-\alpha K_Y\big)(K_X+\alpha K_Y)K_Y}{[\alpha\vartheta_1K_Y-(1-\xi)(K_X+\alpha K_Y)]^2} \tag{C.6}$$

Substituting equations (C.3) - (C.6) into (C.2) gives

$$1-\xi = \frac{(1-\beta)\alpha K_Y}{K_X+\alpha K_Y} \tag{C.7}$$

D Proof of Lemma 2

We can simplify the target firm's value of the option to accept the offer, given by (21), as

$$\tilde{f}^{sale}\big(X(t),Y(t)\big)=\max_{\tau}\mathbb{E}\Big\{e^{-r\tau}\Big[(1-\xi)(K_X+\alpha K_Y)X(\tau)+\big[(1-\xi)(1-\alpha\omega_T-c)+(\lambda-1)\big]K_Y Y(\tau)\Big]\Big\} \tag{D.1}$$

Assume $\tilde{f}^{sale}\big(X(t),Y(t)\big)=Y(t)\tilde{g}^{sale}\big(R(t)\big)$, which satisfies the ordinary differential equation

$$(\frac{1}{2}\sigma_X^2+\frac{1}{2}\sigma_Y^2-\rho\sigma_X\sigma_Y)R^2\tilde{g}^{sale}_{RR}\big(R(t)\big)+(\mu_X-\mu_Y)R\tilde{g}^{sale}_{R}\big(R(t)\big)-(r-\mu_Y)\tilde{g}^{sale}\big(R(t)\big)=0. \tag{D.2}$$

Suppose $\tilde{g}^{sale}\big(R(t)\big)=DR(t)^{\vartheta_1}$, where ϑ_1 is the positive root of equation (9). We can solve $g^{sale}\big(R(t)\big)$ subject to the value-matching and smoothing-pasting conditions

$$\begin{aligned}&D(\tilde{R}^*)^{\vartheta_1}=(1-\xi)(K_X+\alpha K_Y)\tilde{R}^*+\big[(1-\xi)(1-\alpha\omega_T-c)+(\lambda-1)\big]K_Y,\\&\vartheta_1 D(\tilde{R}^*)^{\vartheta_1-1}=(1-\xi)(K_X+\alpha K_Y).\end{aligned} \tag{D.3}$$

Solving the equations above yields

$$\begin{aligned}&\tilde{R}^*(\lambda)=\Big[\big(\alpha\omega_T+c-1\big)+\frac{1-\lambda}{1-\xi}\Big]\frac{\vartheta_1}{\vartheta_1-1}\frac{K_Y}{K_X+\alpha K_Y},\\&D=\frac{(1-\xi)(\alpha\omega_T+c-1)+(1-\lambda)}{\vartheta_1-1}K_Y(R^*(\lambda))^{-\vartheta_1}\end{aligned} \tag{D.4}$$

Hence,

$$\tilde{g}^{sale}\big(R(t)\big)=\frac{(1-\xi)(\alpha\omega_T+c-1)+(1-\lambda)}{\vartheta_1-1}K_Y\Big(\frac{R(t)}{\tilde{R}^*(\lambda)}\Big)^{\vartheta_1}, \tag{D.5}$$

where $R(t)=X(t)/Y(t)$.

E Proof of proposition 3

We can rewrite the optimization function (25) as

$$\tilde{f}^{MA}\big(X(t),Y(t)\big)=\max_{\tilde{\lambda}}Y(t)\Big\{\Big[[(\xi-1)K_X+\xi\alpha\omega_B K_Y]R^*(\tilde{\lambda})+\big[\xi(1-\alpha-c)-\tilde{\lambda}\big]K_Y\Big]\Big(\frac{R(t)}{R^*(\tilde{\lambda})}\Big)^{\vartheta_1}\Big\} \tag{E.1}$$

Suppose

$$\tilde{f}^{MA}\big(X(t),Y(t)\big)=Y(t)\tilde{g}^{MA}\big(R(t)\big), \tag{E.2}$$

where $\tilde{g}^{MA}\big(R(t)\big)$ is given by

$$\tilde{g}^{MA}\big(R(t)\big)=\max_{\tilde{\lambda}}\Big\{\Big[[(\xi-1)K_X+\xi\alpha\omega_B K_Y]R^*(\tilde{\lambda})+\big[\xi(1-\alpha-c)-\tilde{\lambda}\big]K_Y\Big]\Big(\frac{R(t)}{R^*(\tilde{\lambda})}\Big)^{\vartheta_1}\Big\} \tag{E.3}$$

where the threshold function $R^*(\tilde{\lambda})$ given $\tilde{\lambda}$ is (6).

We can solve the optimization problem if the following equation is satisfied:

$$\frac{d\tilde{g}^{MA}\big(R(t)\big)}{d\tilde{\lambda}}=0. \tag{E.4}$$

Therefore, the optimal offered portion satisfies

$$(\vartheta_1-1)[(\xi-1)K_X+\xi\alpha\omega_B K_Y]\frac{dR^*(\tilde{\lambda})}{d\tilde{\lambda}}+\vartheta_1\Big[\xi(1-\alpha-c)K_Y-\tilde{\lambda}K_Y\Big]\frac{1}{R^*(\tilde{\lambda})}\frac{dR^*(\tilde{\lambda})}{d\tilde{\lambda}}+K_Y=0 \quad \text{(E.5)}$$

Solving the equation above yields

$$\tilde{\lambda}^*=(1-\xi)\big(\alpha+c\big)+\xi+\frac{\vartheta_1(\alpha+c)(1-\xi)(K_X+\alpha K_Y)}{(1-\xi)K_X-\alpha[(\vartheta_1+\xi-1)+\vartheta_1\xi(\omega_B-1)]K_Y} \quad \text{(E.6)}$$

Substituting the result above into the equation (22) leads to

$$\tilde{R}^*=\frac{\vartheta_1}{\vartheta_1-1}\Big[\frac{\alpha(\omega_T-1)K_Y}{K_X+\alpha K_Y}+\frac{\vartheta_1(\alpha+c)K_Y}{\alpha[(\vartheta_1+\xi-1)+\vartheta_1\xi(\omega_B-1)]K_Y-(1-\xi)K_X}\Big]. \quad \text{(E.7)}$$

F Proof of proposition 4

The sharing rule is determined by negotiations between the bidder and target. According to the optimization function (28), the sharing-rule satisfies

$$\beta\frac{d\tilde{g}^{MA}\big(R(t);\tilde{\xi}\big)}{d\tilde{\xi}}+(1-\beta)\frac{\tilde{g}^{MA}\big(R(t);\tilde{\xi}\big)}{\tilde{g}^{sale}\big(R(t);\tilde{\xi}\big)}\frac{d\tilde{g}^{sale}\big(R(t);\tilde{\xi}\big)}{d\tilde{\xi}}=0 \quad \text{(F.1)}$$

For the bidder firm, ω_T is unknown until the merger succeeds. Therefore, they will provide an optimal acquisition premium $\tilde{\lambda}$ and estimate the threshold at which the target will decide in a market with perfect information. We can then rewrite the bidder's payoff function as

$$\begin{aligned}\tilde{g}^{MA}\big(R(t);\xi\big)=&\Big\{\big(\xi(K_X+\alpha\omega_B K_Y)-K_X\big)\tilde{R}^*(\xi)+\big(\xi(1-\alpha-c)-\tilde{\lambda}^*(\xi)\big)K_Y\Big\}\Big(\frac{R(t)}{R^*\big(\tilde{\lambda}^*(\xi)\big)}\Big)^{\vartheta_1}\\=&\frac{(\alpha+c)K_Y}{\vartheta_1-1}\Big(\frac{R(t)}{R^*\big(\tilde{\lambda}^*(\xi)\big)}\Big)^{\vartheta_1}\end{aligned} \quad \text{(F.2)}$$

According to the results from (6), this satisfies

$$\frac{1}{R^*\big(\tilde{\lambda}^*(\xi)\big)}\frac{dR^*\big(\tilde{\lambda}^*(\xi)\big)}{d\xi}=\Big[-\frac{d\tilde{\lambda}^*(\xi)}{d\xi}+\frac{1-\tilde{\lambda}^*(\xi)}{1-\xi}\Big]\frac{1}{(1-\xi)\big(\alpha+c-1\big)+1-\tilde{\lambda}^*(\xi)} \quad \text{(F.3)}$$

The first-order differentiation of equation (F.2) with respect to ξ is

$$\begin{aligned}\frac{d\tilde{g}^{MA}\big(R(t);\xi\big)}{d\xi}=&-(\alpha+c)K_Y\frac{\vartheta_1}{\vartheta_1-1}\Big(\frac{R(t)}{R^*\big(\tilde{\lambda}^*(\xi)\big)}\Big)^{\vartheta_1}\frac{1}{R^*\big(\tilde{\lambda}^*(\xi)\big)}\frac{dR^*\big(\tilde{\lambda}^*(\xi)\big)}{d\xi}\\=&\Big(\frac{R(t)}{R^*\big(\tilde{\lambda}^*(\xi)\big)}\Big)^{\vartheta_1}\frac{\vartheta_1}{\vartheta_1-1}\Big[\frac{d\tilde{\lambda}^*(\xi)}{d\xi}-\frac{1-\tilde{\lambda}^*(\xi)}{1-\xi}\Big]\frac{(\alpha+c)K_Y}{(1-\xi)\big(\alpha+c-1\big)+1-\tilde{\lambda}^*(\xi)}\end{aligned} \quad \text{(F.4)}$$

Substituting (F.2) - (F.4) into the optimization function (F.1), we have

$$\begin{aligned}&\frac{\alpha}{\alpha+c}\frac{K_Y}{K_X+\alpha K_Y}\frac{\omega_B}{1-\xi}\Big((1-\xi)(\alpha\omega_T+c-1)+(1-\tilde{\lambda}^*(\xi))\Big)^2\\&=(1-\xi)(\alpha+c-1)+(1-\tilde{\lambda}^*(\xi))+\cfrac{(1-\beta)}{\cfrac{\beta\vartheta_1}{(1-\xi)(\alpha+c-1)+(1-\tilde{\lambda}^*(\xi))}-\cfrac{(1-\beta)(1-\vartheta_1)}{(1-\xi)(\alpha\omega_T+c-1)+(1-\tilde{\lambda}^*(\xi))}}\end{aligned}$$

(F.5)

Suppose

$$\gamma_1 = (1-\xi)(\alpha + c - 1) + (1 - \tilde{\lambda}^*(\xi)) \tag{F.6}$$
$$\gamma_2 = (1-\xi)(\alpha\omega_T + c - 1) + (1 - \tilde{\lambda}^*(\xi)) \tag{F.7}$$

Thus, the optimal sharing-rule satisfies

$$\begin{aligned} &\frac{\alpha}{\alpha + c}\frac{K_Y}{K_X + \alpha K_Y}\frac{\omega_B}{1-\xi}\gamma_2^2 = \gamma_1 + \frac{(1-\beta)}{\frac{\beta\vartheta_1}{\gamma_1} - \frac{(1-\beta)(1-\vartheta_1)}{\gamma_2}} \\ \Rightarrow &\frac{\alpha}{\alpha + c}\frac{K_Y}{K_X + \alpha K_Y}\frac{\omega_B}{1-\xi} = \frac{\gamma_1}{\gamma_2^2} + \frac{(1-\beta)\gamma_1}{\beta\vartheta_1\gamma_2^2 - (1-\beta)(1-\vartheta_1)\gamma_1\gamma_2} \end{aligned} \tag{F.8}$$

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